UUCMS. No.

B.M.S COLLEGE FOR WOMEN, AUTONOMOUS

BENGALURU – 560004

SEMESTER END EXAMINATION – SEPT/OCT-2023

M.Sc in Mathematics – 2nd Semester

TOPOLOGY–II

Course Code: MM203T **QP Code: 12003**

Duration: 3 Hours **Max. Marks: 70**

Instructions: 1) All questions carry equal marks.
2) Answer any five full questions.

1. a) Prove that the continuous image of a compact space is compact.
   b) Prove that a Hausdorff space is locally compact if and only if each point has a neighbourhood whose closure is compact.
   c) Prove that every sequentially compact space is countably compact.

(4+5+5)

2. a) Define FAS and SAS. Prove that SAS is both hereditary and topological property.
   b) If every countable open cover of (X, τ) has a finite sub cover, then prove that X is countably compact.

(8+6)

3. a) Define projections on the product space $X \times Y$ and show that they are continuous and open maps.
   b) Prove that $X \times Y$ is second countable if and only if X and Y are second countable.

(7+7)

4. a) Define T_0 space. Prove that in a T_0- space the closure of distinct points are distinct and
   conversely.
   b) Prove that T_1 - space is both hereditary and topological property.

(7+7)

5. a) Prove that a regular $T_0 -$ space is a $T_3 -$ space.
   b) Prove that every compact subset of a Hausdorff space is closed.
   c) Prove that (X, τ) is normal if and only if given any open set G and a closed set $F \subseteq G$,
   there exists an open set G^* such that $F \subseteq G^* \subseteq \overline{G^*} \subseteq G$.

(4+5+5)

6. a) Show that a regular Lindelöf space is normal.
   b) Prove that every metric space is normal.

(7+7)

7. a) State and prove Urysohn's Lemma.
   b) Prove that a space is completely normal if and only if every subspace is normal.

(8+6)

8. a) Prove that every metric space is completely normal.
   b) Prove that a paracompact Hausdorff space is normal.

(7+7)

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